STRUCTURE, PROPAGATION, AND REFLECTION OF SHOCK WAVES IN A MIXTURE OF SOLIDS (THE HYDRODYNAMIC APPROXIMATION)

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Problems of mathematical modeling of the behavior of mixtures of various materials at high pressure are generated in calculating the action of products of high-energy detonation explosives on heterogeneous materials, porous media, and so on.

To describe these effects within the hydrodynamic approximation of mechanics of heterogeneous media, corresponding mathematical models were suggested in [1, 2] within the two-pressure approximation. In the case of nonequal phase pressures some closure relation is required, such as a condition of pressure proportionality $p_1 = kp_2$, $k = \text{const}$ (including $k = 1$). A method of model closure in pressure, different from that mentioned, was suggested in [3]. It is based on postulating equations of $m_2$-transport for the bulk concentration of the second phase with a source term. An expression for the source term in the equations of $m_2$-transport is derived in [4], and a closure model is given for two solids.

According to the mathematical model suggested, the calculation of the structure of the shock wave (SW) in a saturated porous medium (water and sand) is carried out in [1] for a single-pressure mixture. Problems of solution existence and uniqueness have not been discussed in detail. A review is provided in [5] of studies on SW structures in mixtures of two solid materials within the two-velocity, single-pressure, barotropic approximation. The existence of four SW types is shown in [6] on the basis of qualitative arguments for this flow. In [7] were investigated problems of existence and uniqueness of solutions of the type of traveling waves for a mixture of Clapeyron gases within the two-velocity, two-temperature approximation, while similar problems were treated in [8] for a single-velocity, two-pressure barotropic flow of a gas–fluid mixture.

It is interesting to investigate the SW structure in a mixture of two solids within the hydrodynamic approximation including the differences in the phase velocities and pressures, as well as formation of different types of SWs from initial given step-function shapes and SW reflection from a rigid wall.

1. Stationary Flow. The equations of [9], describing flow of the traveling wave type in an attached coordinate system, are

$$\begin{gathered}
\rho_i u_i = c_i, \quad i = 1, 2, \quad p + c_1 u_1 + c_2 u_2 = c_3, \\
p_i = a_i^2 (\rho_i / m_i - \rho_{ii,0}), \quad i = 1, 2, \quad p = m_1 p_1 + m_2 p_2, \\
c_2 \dot{u}_2 + m_2 \dot{p}_2 + (p_2 - p_1) \dot{m}_2 = -R, \\
R = -\frac{\rho_2 c_D}{\tau_{St}} \frac{\mathrm{Re}}{24} m_1 (u_1 - u_2), \quad \dot{m}_2 = -\varkappa = -\frac{(p_1 - p_2)}{\mu_2 u_2}.
\end{gathered} \tag{1.1}$$

Here $\rho_i = m_i \rho_{ii}$ is the mean density, $m_i$, $\rho_{ii}$ are the bulk concentration and true density, $p_i$ is the partial pressure, $u_i$ are the velocities of the different phases, zero denotes the initial state, $c_i$ are the values of the corresponding discharges at the initial point, R is the interphase interaction force, $c_D$ is the resistance coefficient, Re is the Reynolds number, and $\tau_{St}$ is the Stokes velocity relaxation time. The system (1.1) must satisfy stationary boundary conditions at $\pm\infty$ for the vector solution $\Phi = \Phi(\rho_1, \rho_2, u_1, u_2, \dot{m}_2)$:

$$\Phi \to \Phi_{0,\,\text{K}}, \ \dot{\Phi} \to 0 \text{ for } x \to \pm\infty. \tag{1.2}$$

The propagation problem of a stationary SW in a mixture of two solid materials reduces then to solving the boundary value problem (1.1), (1.2) in the region $(-\infty, +\infty)$.

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 10-18, July-August, 1992. Original article submitted January 28, 1991; revision submitted June 19, 1991.

Two relaxation processes occur in the mixture: velocity compensation of the components with characteristic time scale $\tau_{St}$ and variation of the bulk concentration of metal particles, generated by the pressure difference of phases with scale $\tau_{m2} = 2r\rho_{22,0}a_2/(\rho_{11,0}a_1^2)$ (r is the particle radius, and for the viscosity of the second medium we adopt the estimate $\mu_2 = 2\rho_{22}a_2 r$). One can also define a characteristic time of perturbation propagation in the mixture $\tau_g = x_0/a_2$ ($x_0$ is a characteristic length).

A mixture flow in which $\tau_{St}/\tau_g \ll 1$, $\tau_{m2}/\tau_g \ll 1$ will be called total equilibrium or equilibrium, $\tau_{St}/\tau_g \ll 1$, $\tau_{m2}/\tau_g \sim 1$ – equilibrium-frozen, $\tau_{St}/\tau_g \sim 1$, $\tau_{m2}/\tau_g \ll 1$ – frozen-equilibrium, and $\tau_{St}/\tau_g \gg 1$, $\tau_{m2}/\tau_g \gg 1$ – frozen.

The first type of motion is characteristic of equal phase velocities and pressure, the second – equal velocities but different pressures [9], the third – different phase velocities and equal pressures, and the fourth – different velocities and pressures. We further determine the total equilibrium sound velocity $c_e^2 = dp/d\rho$, where $p = p_1(\rho_1, m_2^e(\rho)) \equiv p_2(\rho_2, m_2^e(\rho))$, $\rho_1 = (1-\alpha)\rho$, $\rho_2 = \alpha\rho$, $\alpha = \rho_{20}/\rho_0$.

It is easily verified that

$$c_e^2 = c_{ef}^2 + (S_1 + p_2 - p_1)\, dm_2^e/d\rho.$$

Here $dm_2^e/d\rho = R_1/P_1$; $S_1 = m_2\partial p_2/\partial m_2 - m_1\partial p_1/\partial m_1$; $\xi_i = \rho_i/\rho$ $(i = 1, 2)$; $R_1 = \xi_1\partial p_1/\partial\rho_1 - \xi_2\partial p_2/\partial\rho_2$; $P_1 = \partial p_2/\partial m_2 + \partial p_1/\partial m_1$. The function $m_2^e(\rho)$ is found from the condition of pressure equality. For the second flow type in the mixture one defines the equilibrium-frozen sound velocity $c_{ef}$:

$$c_{ef}^2 = \partial p/\partial\rho\Big|_{\substack{u_1=u_2\\ p_1\neq p_2}} = \xi_1 a_1 + \xi_2 a_2,$$

and for the third – the frozen-equilibrium:

$$c_{fe}^2 = \partial p/\partial\rho\Big|_{\substack{u_1\neq u_2\\ p_1=p_2}} = (m_2\,\partial p_2/\partial\rho_2\partial p_1/\partial m_1 + m_1\,\partial p_1/\partial\rho_1\partial p_2/\partial m_2)/P_1.$$

We note that there exist two frozen sound velocities: $a_1$ and $a_2$, whose maximum $a_2$ will be called the totally frozen (or frozen) sound velocity.

In the equilibrium flow of the mixture $[p_1(\rho) = p_2(\rho)]$ one can find a dependence of the bulk concentration of phases on the mean density

$$P(\rho, m_1) = cum_1^2 - m_1(cu + c_1c_{ef}^2) + \xi_1 c_1 = 0$$

$[u = c_1/\rho,\ c = 1 - a\rho_{22,0}/\rho_{11,0},\ a' = (a_2/a_1)^2]$. It is shown that the discriminant of this equation is positive, therefore both branches of the solution are real. The branch $m_1 = m_1^e(\rho)$, corresponding to the minus sign in the expression for the root of the quadratic equation, has a physical meaning, since it passes through the initial point $(m_{10}, \rho_0)$.

The equation of state is in this case

$$P = \xi_1\rho/m_1^e(\rho) - 1.$$

The set of states behind the SW front can be found initially by means of the Hugoniot equilibrium adiabat. It seems that in this case there exist two final states: $u_K^+$, $u_K^-$, defined as the solutions of the quadratic equation

$$u_K^2 + bu_K + \tilde{c} = 0,$$

where $\tilde{c} = ((1+y)c_{ef} - \xi_1 c)/(c_1 M_{10})$; $b = -(2 + y - c)/c_1$; $y = \rho_0 M_{10}^2$. Indeed, the discriminant of the given equation can be represented as a quadratic polynomial in y, which is positive for all physical initial data. In this case we have for the Mach numbers of the solutions found for the equilibrium sound velocity $c_e$

$$M_{e,K} = u_K/c_{e,K},\quad c_e^2 = \xi_1(cm_1^e - \xi_1\rho)/(m_1^e(c(m_1^e)^2 - \xi_1\rho)),$$

the following estimates $M_{e,K}^+ > 1$, $M_{e,K}^- < 1$.

To determine the conditions behind a frozen SW with $m_1 = m_{10}$ one can use the Hugoniot adiabat of the frozen flow, which we rewrite as

$$F(u_1, u_2) = \frac{(u_2 - u_0)(u_2 - \tilde{u}_2)}{u_2} - \frac{\xi_1}{\xi_2}\frac{(u_1 - u_0)(u_1 - \tilde{u}_1)}{u_1} = 0 \qquad (1.3)$$

($\tilde{u}_1 = 1/u_0$, $\tilde{u}_2 = a/u_0$). It is hence seen that three states correspond to the equilibrium point ($u_0$, $\tilde{u}_2$), ($\tilde{u}_1$, $u_0$), ($\tilde{u}_1$, $\tilde{u}_2$), i.e., the conditions behind the frozen SW front are also found nonuniquely.

We finally obtain conditions behind the front of an equilibrium-frozen SW, i.e., when $u_1 = u_2$, $m_1 = m_{10}$, $p_1 \neq p_2$. In this case the velocity behind the front is $\tilde{u} = \xi_1\tilde{u}_1 + \xi_2\tilde{u}_2 = c_{ef}^2/u_0$. The solution of the boundary value problem (1.1), (1.2) reduces to solving the system of differential equations

$$\frac{du_1}{dx} = \frac{u_1}{\rho_1}\,\frac{R + \varkappa m_1 \partial p_1/\partial m_2}{u_1^2 - a_1^2} = A(u_1, u_2); \tag{1.4}$$

$$\frac{du_2}{dx} = \frac{u_2}{\rho_2}\,\frac{-R + \varkappa(m_2 \partial p_2/\partial m_2 + p_2 - p_1)}{u_2^2 - a_2^2} = B(u_1, u_2) \tag{1.5}$$

with corresponding boundary conditions.

It was shown in [9] that for $\tau_{St}/\tau_g \ll 1$, $\tau_m/\tau_g \sim 1$ frozen and disperse SWs can propagate in the mixture, and criteria of pressure nonmonotonicity in the second phase were found. We investigate the flow types in the mixture for $\tau_{St}/\tau_g \sim 1$, $\tau_{m2}/\tau_g \gg 1$. In this case one can show the solvability of the boundary value problem for Eq. (1.4), augmented by the integral (1.3). From Eq. (1.3) we explicitly express $u_2$ as a function of $u_1$. The range of definition of the function $u_2(u_1)$ consists of three subregions. In the first ($0 < u_1 < u_1^3$) the function $u_2(u_1)$ is nonunique (when $u_1 \to 0$ one of the branches tends to zero, and the second – to $\infty$); in the second ($u_1^1 \leqslant u_1 \leqslant u_1^2$) it is a closed curve, and in the third ($u_1 > u_1^4$) – it is also nonunique [here the quantities $u_1^i$ ($i = 1, 4$) depend on the initial parameters of the mixture]. For $u_1^3 < u_1 < u_1^1$, $u_1^2 < u_1 < u_1^4$ the function $u_2(u_1)$ is undefined. The straight line $u_2 = u_1$ intersects the function $u_2(u_1)$ described at two points: ($u_0$, $u_0$), ($u_K$, $u_K$). To find $u_K = c_{ef}^2/u_0$ one can use the Hugoniot adiabat in equilibrium flow in velocities, nonequilibrium flow in pressures with $u_0 > c_{ef}$. The results of estimating velocity values at the final point for various initial data $u_0$, $\xi_1$ are given in Table 1, in which $\tilde{\xi} = (u_0^2 - a)/(1 - a)$, and

$$\xi_* = \sqrt{a}(u_0 - \sqrt{a})/(1 - a), \quad \xi_{**} = (u_0 - a)/(1 - a).$$

We investigate the sign of $\lambda_1 = dA(u_1, u_2(u_1))/du_1$ at the initial and final flow points [$A(u_1, u_2)$ is defined by (1.4)]. Using the representation

$$\frac{du_2}{du_1} = -\frac{c_1(u_1^2 - 1)u_2^2}{c_2(u_2^2 - 1)u_1^2}$$

and putting for simplicity $c_D = 24/Re$, for $u_1 = u_2 = u$ we obtain

$$\lambda_1 = -\frac{\rho_2(u^2 - c_{ef}^2)u}{\rho_1(u^2 - 1)(u^2 - a)\tau_{St}\xi_2}.$$

Let $\Phi_0 = (u_0, \xi_1)$ be the vector of initial data, and let $\lambda_{10}$, $\lambda_{1K}$ be the $\lambda_1$ values at the initial and final flow points. We then have

<u>Statement 1.</u> If $\Phi_0 \in \mathrm{I}_1$, then $\lambda_{10} < 0$, $\lambda_{1K} < 0$; if $\Phi_0 \in \mathrm{I}_2$, then $\lambda_{10} < 0$, $\lambda_{1K} > 0$; if $\Phi_0 \in \mathrm{II}_1, \mathrm{III}_1, \mathrm{IV}$, then $\lambda_{10} < 0$, $\lambda_{1K} < 0$; and if $\Phi_0 \in \mathrm{II}_2, \mathrm{III}_2$, then $\lambda_{10} > 0$, $\lambda_{1K} > 0$.

On the basis of this statement we omit the flow types in the regions mentioned. The qualitative flow pattern can be represented by Fig. 1. The points $O(u_0, u_0)$, $B_1(u_0, \tilde{u}_2)$, $C_1(\tilde{u}_1, \tilde{u}_2)$, $A_1(\tilde{u}_1, u_0)$ on the closed curve correspond to conditions in frozen compression and dilatation waves. The straight line $u_2 = u_1$ passes through the points $O(u_0, u_0)$, $K(u_K, u_K)$. Transforming from the point O to the points $A_1$, $C_1$, $B_1$ with a discontinuity, the flow further relaxes to the points $D_1$, $D_2$, in which the velocity of the second phase reaches the sound velocity. Following this we have flow locking. Here, following [10], one can introduce L – the maximum sample length of a heterogeneous medium in which the mixture flow takes place. For a sample length $\ell < L$ subsonic flow is realized at the final point, for $\ell = L$ we have sonic flow in the second phase, and for $\ell > L$ the flow is nonreal. Using the Lax stability conditions, it can be shown that the flows $C_1 \to D_1$, $B_1 \to D_2$ are unstable (dilatation SWs in both phases and in the second phase, respectively). For $A_1 \to D_1$ the SW is stable in the first phase and is terminated at a final distance L from the wave front. At this point we have peaking of the quantity $du_2/dx$. Following S. A. Khristianovich, this flow can be interpreted as a fractal wave.

TABLE 1

| $\xi_1$ | $u_0$: $1 < u_0 < c_{ef}$ | $c_{ef} < u_0 < \sqrt{a}$ | $\sqrt{a} < u_0 < a$ | $u_0 > a$ |
|---|---|---|---|---|
| $\xi_1 > \xi_{**}$ | No solutions | $\mathrm{II}_1$ $u_K < 1$ | $\mathrm{III}_1$ $u_K < 1$ | IV $u_K < 1$ |
| $\tilde{\xi} < \xi_1 < \xi_{**}$ | » » | $\mathrm{II}_2$ $c_{ef} > u_K > 1$ | $\mathrm{III}_2$ $1 < u_K < c_{ef}$ | |
| $\xi_* < \xi_1 < \tilde{\xi}$ | $\mathrm{I}_1$ $c_{ef} < u_K < \sqrt{a}$ | No solutions | | |
| $0 < \xi_1 < \xi_*$ | $\mathrm{I}_2$ $u_K < \sqrt{a}$ | » » | | |

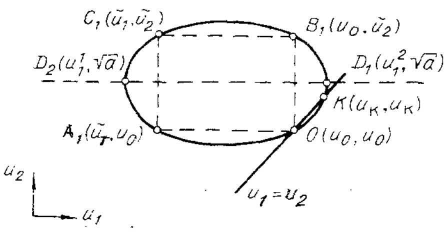


Fig. 1

The flow in region $\mathrm{I}_2$ is treated similarly. Here the final point was shifted above the line $u_2 = \sqrt{a}$. The flow is stable, starting at the point $A(\tilde{u}_1, u_0)$ and terminating at the point $D_1(u_1^1, \sqrt{a})$ in the regime with peaking of $du_2/dx$.

During flow in the region $\mathrm{II}_1$ the function $u_1(x)$ acquires at $x \to \mp\infty$ the values $u_0$, $u_K$ with a turning point at $u_1 = 1$. In this case $u_2$ varies continuously and uniquely from $u_0$ to $u_K$. To eliminate the nonuniqueness in the velocity of the first component at the point in which the equality $u_2 = u_K$ is reached, a discontinuity is introduced in the first phase. The phase velocities are equated behind it: $u_1 = u_2 = u_K$. We note that equilibrium is achieved at a finite distance by means of a tail discontinuity.

The solution is region $\mathrm{II}_2$ with $\lambda_0 > 0$, $\lambda_K < 0$, being continuous, asymptotically acquiring initial and final values at $\mp\infty$, is a disperse SW. Let $u_0 = \alpha c_{ef} + (1 - \alpha)\sqrt{a}$, $\alpha \in (0, 1)$, where $c_{ef} = \left(\alpha + \sqrt{\alpha^2 + 4(1-\alpha)\sqrt{\alpha}}\right)/2$, $\xi_1 = (c_{ef} - a)/(1 - a)$, $M_1 = \xi_1\rho_{22}/(\xi_2 + \xi_1\rho_{22})$, in which case $u_K = 1$. An unstable point of intermediate equilibrium $u_1 = u_2 = u_K = 1$ $[du_1/dx(u_K) \neq 0$, $du_1/dx(u_K) \neq \infty$, $du_2/dx(u_K) = 0]$ is reached in this case in the solution. The velocity of the second phase increases to $\sqrt{a}$ during transition through this point, and the velocity of the first phase decreases to $u_1^1$. The given regime has transition points through both phase sound velocities with peaking of $du_1/dx$ in the final interval. It is the boundary between a disperse SW with a tail discontinuity and a totally disperse SW.

It is similarly shown that in regions $\mathrm{III}_1$, IV there exists a flow with a two-wave configuration. At the tip of the SW there is a jump in the second phase $(u_0, u_0) \to (u_0, \tilde{u}_2)$, and then a velocity relaxation zone occurs until $u_1$ is comparable with $u_K$. At the same point there occurs a SW transition to a final point in the first phase. The flow in region $\mathrm{III}_2$ is a frozen SW in the first phase in the tip portion, augmented by a velocity relaxation zone in both phases. The regimes $\mathrm{III}_1$, $\mathrm{III}_2$ are separated by a flow with $m_1 = m_{10}$, for which the solution is a frozen SW in the first phase, augmented by a region of continuous flow, in which a transition is realized through both sound velocities. This flow exists in a regime with peaking $du_2/dx$ at the final length of the medium L.

On the basis of this discussion we formulate

Statement 2. A solution of problems (1.1), (1.2) exists in the class of: 1) frozen SWs, augmented by a relaxation zone with peaking of $du_2/dx$ in the final flow region with $(u_0, \xi_1) \in \mathrm{I}_1$, $\mathrm{I}_2$; 2) disperse SWs with a tail discontinuity in the first phase at $(u_0, \xi_1) \in \mathrm{II}_1$, and totally disperse SWs at $(u_0, \xi_1) \in \mathrm{II}_2$; and 3) frozen SWs in the second phase (the tip of the discontinuity), the boundaries through a relaxation zone with a tail discontinuity in the first phase, with $(u_0, \xi_1) \in \mathrm{III}_1$, IV, and totally frozen SWs at $(u_0, \xi_1) \in \mathrm{III}_2$.

For $\xi_1 = \xi_{**}$ the flow in regions $\mathrm{II}_1$, $\mathrm{II}_2$ exists in the form of a disperse wave with two sonic points and peaking of $du_2/dx$ on a semibounded interval. If $\xi_1 = \xi_{**}$ in the region $\mathrm{III}_1$, the relaxation flow has the form of a frozen SW with two sonic points on a semibounded interval with peaking of $du_2/dx$. A full discussion of the proof of existence of the flow regimes of the mixture is given in [15].

2. Nonstationary Flow. The equations describing flow of a mixture in the nonstationary one-dimensional case are

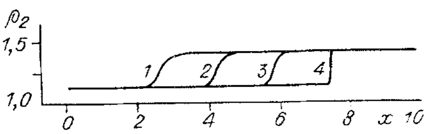


Fig. 2

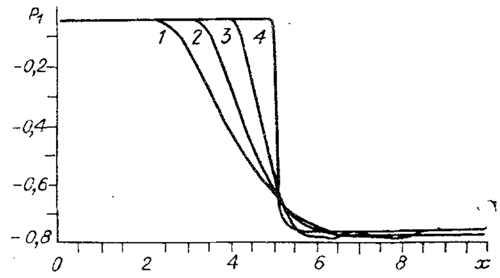


Fig. 3

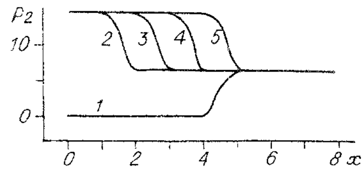


Fig. 4

$$
\begin{gathered}
\partial\rho_i/\partial t + \partial\rho_i u_i/\partial x = 0,\ i = 1, 2, \\
\rho_1(\partial u_1/\partial t + u_1\partial u_1/\partial x) + m_1\partial p_1/\partial x = -\rho_1(u_1 - u_2)c_D\mathrm{Re}/24/\tau_{\text{ст}} = R, \\
\rho_2(\partial u_2/\partial t + u_2\partial u_2/\partial x) + m_2\partial p_2/\partial x = -R + (p_2 - p_1)\partial m_1/\partial x, \\
\partial m_2/\partial t + u_2\,\partial m_2/\partial x = -(p_1 - p_2)/\mu_2,\quad p_i = a_i^2(\rho_i/m_i - \rho_{ii,0}),\quad i = 1, 2.
\end{gathered} \tag{2.1}
$$

As initial data we take the parameter distribution in the form of one of three possible types of stationary SWs. The problem is solved by the coarse particle method, investigating the following problems for this case.

Problem 1. Propagation of Disperse and Frozen SWs in Space. Let $u_{10} = u_{20} = 0$, $D = -3.2$, $m_1 = 0.5$ (the various quantities are rendered dimensionless according to [9]). In this case the flow is a totally disperse SW at t = 0. Since the flow is subject to constant equilibrium parameter values at the right boundary of the region (for the velocities these are $u_i = u_K$, i = 1, 2), a stationary disperse SW propagates to the left. The formation process of a disperse SW was investigated from initial data of a step type. In this case the flow parameters to the left and to the right of the discontinuity point satisfied the equilibrium Hugoniot adiabatic curve. As seen from the density profiles of the second phase shown in Fig. 2, the discontinuity occurring at the moment of time t = 0 (curve 4) is smoothed at t = 0.5 (curve 3), and a disperse SW propagates at t = 1, 1.5 (curves 2, 1, respectively).

Flow versions with $u_{10} = 0$, $u_{20} = 0$, $D = -3.5$, $m_1 = 0.75$ have been treated similarly. A frozen SW with single-wave structure is realized at t = 0, with the flow being continuous in the first phase with a discontinuity in the second. The leading discontinuity is somewhat blurred due to the effect of the viscosity approximation, and the wave propagates stably with a velocity near D = −3.5.

Let $u_{10} = 0$, $u_{20} = 0$, $D = -6.5813$, $m_1 = 0.75$. The parameters of the first phase are continuous at the tip of the wave have a tail discontinuity at t = 0, while in the second phase the SW is located at the tip of the flow. In the given case one observes stable stationary propagation of the wave configuration investigated. We note that the flow in the second phase with the leading discontinuity is formed more quickly than in the first.

The behavior of a mixture in which flow of the type of SW dilatation is realized at the initial moment of time is of interest. The presence of an unstable discontinuity in the second phase, augmented by a velocity relaxation zone to the final equilibrium state, is characteristic of it. The flow in the first phase is continuous. This flow type is realized for $u_{10} = 0$, $u_{20} = 0$, $D = -1.5$. Since the relative sound velocity in the final equilibrium state $u_K - D$ is higher than the sound velocity in the second phase, the relaxation zone is continuously adjacent to the final state. The flow pattern generated for the pressure in the first phase is given in Fig. 3. The unstable flow configuration at t = 0 (curve 4) decomposes into two dilatation waves (DW), propagating to the right and to the left (curves 1-3 correspond to t = 1.2, 0.8, 0.4). The DW propagating to the right reduces the pressure in the components, and a constant flow zone of the mixture is formed behind it. The given configuration results from the fact that the pressure discontinuity is concentrated at the point x = 5. A discharge starts with the decomposition, which reduces the pressure $p_0 = 0$ and leads to motion of the medium to the right of the discontinuity. Its velocity, however, is high, and the points of the mixture shown in the motion (located left of the point x = 5) are not subject to the pressure of the right half, since the velocity of particle "escape" is quite high in the given case. In the given case the situation is similar to the problem of configuration B described in [11] of discontinuity decay, if in the latter one carries out the variable replacement $u = -u_{10} + u$, $x = x - u_1 t$ ($u_1 < 0$ is the gas velocity left of the discontinuity). Calculations carried out over a long time with removed boundaries of the computational region have shown that equilibrium in the flow velocities ($u_1 = u_2 = 2.8$) is formed in the mixture as $t \to \infty$, while the pressure components are nonequal and are close to the pressure values at the profile intersection point at different moments of time.

Problem 2. SW Reflection from a Wall in the Mixture. This problem has attracted the attention of researchers (a bibliography can be found in [12, 13]). We note the study [14], where the conditions determining the type of reflection are written down for incident frozen and disperse waves in a mixture of gas and solid particles. A verification of these conditions is provided on the basis of calculations within a mixture model without inclusion of bulk particles (the Kligel–Nickerson model). The calculations have verified the three formulated types of SW reflection.

We dwell now on the problem of SW reflection from a rigid wall in the case of two compressible gases. We assume that a compression SW with constant amplitude propagates from right to left in the resting mixture. The flow ahead of the SW at $t = 0$ is in equilibrium with the parameters $p_i = p_0 = 0$, $u_i = 0$, $\rho_i = \rho_{i0}$, $i = 1, 2$. The flow parameters behind the incident SW are marked by the subscript K, and the subscript R refers to the reflected SW ($D_K$, $D_R$ are the velocities of incident and reflected SWs). Behind the front of the incident SW the parameters of the mixture acquire equilibrium in the velocity values $u_i = u_K$, $p_i = p_{iK}$, $i = 1, 2$. The conservation laws are in this case

$$-\rho_0 D_K = \rho_K (u_K - D_K), \quad \rho_0 D_K^2 = p_K + \rho_K (u_K - D_K)^2, \quad -\rho_R D_R = \rho_K (u_K - D_R), \quad p_R + \rho_R D_R^2 = p_K - \rho_R D_R (u_K - D_R),$$

where $p_K = m_1 p_{1K} + m_2 p_{2K}$; $\rho_0 = \rho_{10} + \rho_{20}$; $\rho_K = \rho_{1K} + \rho_{2K}$; $p_K = c_{ef}^2(\rho_K - \rho_0)$; $p_R = c_{ef}^2(\rho_R - \rho_0)$.

Following simple transformations for the determination of $D_R$, we obtain the equation $D_R^2 - D_R u_K - c_{ef}^2 = 0$, having solutions $D_{2+} = D = -u_0$, $D_{2-} = -c_{ef}^2/D$. The first of them determines the velocity of the incident SW, and the second – the reflected one. We write down equations for the relative velocities of the mixture ahead of and behind the front of the reflected SW, respectively: $u_K' = u_K - D_{2-} = D$, $v_K' = -D_{2-} = c_{ef}^2/D$. On the basis of these expressions and the statements proved above for the incident disperse SW it is shown that the incident disperse SW is reflected by a wave of the same type, i.e., for $(u_0, \xi_1) \in \mathrm{II}_1$ – a disperse SW with a tail discontinuity in the first phase, and $(u_0, \xi_1) \in \mathrm{II}_2$ – a totally disperse SW. For $(u_0, \xi_1) \in \mathrm{III}_1$ the incident SW is reflected by a wave of similar type, and the incident frozen wave of single-wave structure is reflected by a frozen wave of the same structure. If $(u_0, \xi_1) \in \mathrm{III}_2$, the incident and reflected waves have a two-wave structure.

We describe several numerical experiments, carried out for this problem. As initial conditions we select a flow of the type of a completely disperse SW, corresponding to $u_{i0} = 0$, $D_K = 3.2$, $m_{10} = 0.5$, $i = 1, 2$. At $t = 0$ the tip of the SW is concentrated at $x = 4$ (by the tip of the SW we imply the point in space at which the flow parameters at $\varepsilon = 0.01$ differ from the values at the front of the disperse SW). Figure 4 shows pressure profiles in the second phase at $t = 0$, 0.2, 0.24, 0.28, 0.32 (curves 1-5). As is seen, the reflected SW is also disperse. In this case one must notice the good transfer of parameters behind the front of the reflected SW: the analytic solution gives $\rho_{2,R}/\rho_{2,0} = (D/c_{ef})^4 = 1.545$, and the numerical solution gives 1.540.

The behavior of the phase velocities is interesting. Thus, for propagation of a SW from the left toward the rigid wall the profile of phase velocities is a curve with sharp variation in the leading portion of the wave and is smoother in the rear. Figure 5a shows the behavior of $u_2$ at $t = 0$, 0.4, 0.8, 1.2, 1.6 (curves 1-5), and Fig. 5b – at $t = 0$, 2, 2.4, 2.8, 3.2 (curves 1-5). The wave velocity is in this case $D_R = 3.2$. The wave traverses the distance $\ell = 4$ after $\Delta t = 1.25$ (curve 1 of Fig. 5a). It is seen that the pattern of velocity change with wave thickness is totally reversible. In the leading part the velocity variation is primarily sharp, and in the trailing part it is smoother; the velocity of the reflected disperse SW (obtained analytically) $D_R = -c_{ef}^2/D = 2.57$ is close to the calculated $\tilde{D}_R = 2.58$.

We note that the pressure and density of the first phase near the wall slightly exceed the values determined analytically. This is, obviously, related to the fact that here one deals with interactions between the incident and reflected waves. Therefore the mixture does not reach a homogeneous state at the final time.

We further consider reflection of a frozen SW from the wall ($m_1 = 0.75$, $D = 3.5$, $u_{i0} = 0$, $i = 1, 2$) in the case of single-wave configuration (the SW tip in the second phase, with continuous flow in the first). The calculation results for the pressure of the first phase are shown in Fig. 6 (curves 1-5 correspond to $t = 3.2$, 3.6, 4.0, 4.4, 4.8). In this case

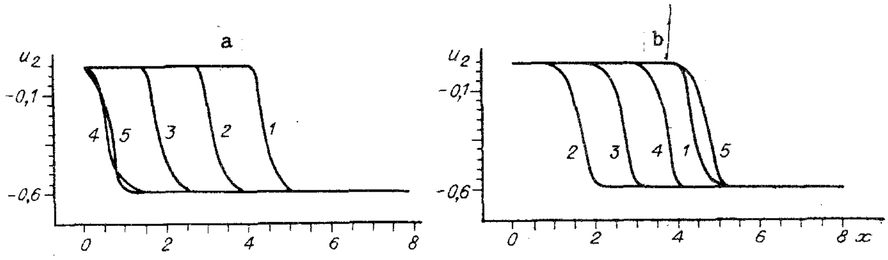


Fig. 5

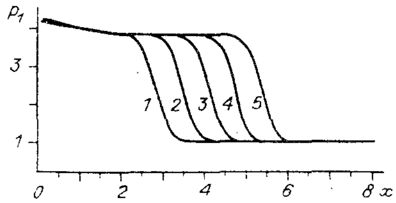


Fig. 6

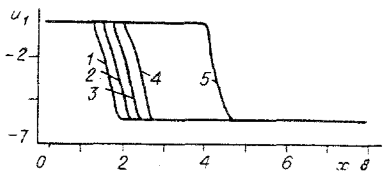


Fig. 7

pressures are generated during impact of the first phase with the solid wall at the first moments of time following impact, which are substantially higher than in the state behind the reflected wave. The pressure at the wall decreases to the extent of wave removal from the wall. However, calculations carried out to a distance of six calibers (a caliber is the ratio of the linear size to the width of the frozen SW) have shown that $p_2(0, t)$ has not yet reached a steady state. The pressure in the second phase, where a discontinuity exists in the tip of the wave, increases at the wall up to its value behind the front. In this case the $p_1(0, t)$ value is established substantially more quickly than in the second phase. At a distance of 1.5 calibers already the pressure in the first phase is near the limiting value. Naturally, the phase densities behave similarly. As in the preceding case, the solution was reflected symmetrically.

Consider the reflection of a frozen SW with a two-wave configuration ($u_{10} = u_{20} = 0$, $p_{10} = p_{20} = 0$, $m_1 = 0.75$, $D_K = 6.5813$), when the flow is continuous in the first phase at the tip of the wall, while a discontinuity occurs at the tail and relaxation with the subsequent zone takes place in the second phase at the tip of the discontinuity. The calculation results for the velocity of the first phase are shown in Fig. 7 (the initial data are line 5, and the solutions at t = 2.4, 2.7, 3, 3.3 – lines 1-4). It is seen that following reflection from the wall the frozen SW remains a wave of the same type, moving with velocity $D_R = -0.83$. The tail discontinuity in this phase is reproduced in the numerical calculations in a worse manner than at the tip: nonphysical oscillations of small amplitude are generated behind its front. The leading discontinuity in the second phase is reproduced quite satisfactorily. At a distance approximately equal to 5 calibers from the wall the reflected wave already propagates in the stationary regime. There is a clear distinction between the tip of the discontinuity and the subsequent zone of stationary values.

Thus, in this study we formulated and proved statements concerning types of stationary waves in two-velocity two-pressure mixtures of solids (the hydrodynamic approximation). The stability of propagation of compression waves in the mixture was shown numerically. A similar B configuration of gas-dynamic decomposition decay of the discontinuity [11] is found in investigating the evolution of instability of a dilatation SW in the mixture. It has been shown analytically and numerically that during SW incidence at the wall its type is conserved.

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# NUMERICAL INVESTIGATION OF FLAME PROPAGATION AND EXTINCTION IN A VERTICAL CHANNEL

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One of the most important problems in the theory of combustion limits is the role of natural convection in the extinction process. It is well known that the direction of flame propagation strongly influences the combustion concentration limits: they are narrower in downward propagation than when the flame travels upward.

A hypothesis explaining the mechanism of extinction of a flame as it propagates down through a vessel from the upper wall has been advanced in [1]. The authors suggested that, because of the cooling of hot reaction products by the vessel walls behind the flame front, free-convection flows develop, causing additional heat loss from the combustion zone and extinguishing the flame. Subsequent studies have confirmed the correctness of the hypothesis and led to the creation of approximate theoretical models of this phenomenon [2-6].

The formation of convective vortices behind a combustion front and their influence on flame propagation and shape in ignition from above have been studied in [7, 8]. An experimental study of the influence of gravity on flame propagation in a tube led Strehlov et al. [9] to conclude that extinction in weightlessness and in downward motion of the front is explained by heat transfer to the walls.

A considerable number of papers have been devoted to determining the critical conditions of combustion in ignition from below. Experiments [10] have shown that in a tube, the limiting flame propagation velocity opposite to the gravitational vector is determined by the ascent velocity of hot reaction products, which depends on the tube diameter and the free-fall acceleration; extinction was assumed to occur if the combustion velocity is less than the upwelling velocity of burned gas. In [11], on the basis of a similar extinction hypothesis, the fundamental limiting flame velocity was calculated in the case of ascent of the burning

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 18-26, July-August, 1992. Original article submitted May 23, 1991.

 0021-8944/92/3304-0494$12.50 